2m+1 Point Estimate Method for Probabilistic Harmonic Power Flow

Guangzheng Yu
Department of Electrical Engineering
Wuhan University
Wuhan, China
wdtbygz@live.com

Tao Lin
Department of Electrical Engineering
Wuhan University
Wuhan, China
1848353649@qq.com

Abstract—Distributed generators (DGs) access to smart distribution network bring harmonic current injection to power system and the distribution of injected harmonic current is uncertain. Probabilistic Harmonics Power Flow (PHPF) is a tool to analyze influence of harmonic current uncertainty to the grid so that a method based on the 2m+1 point estimate method (PEM) is proposed to solve the problem. This approach construct three estimate points by high order moments and the establishment analytic form relationship between input and output variables is avoided. Based on the high order moment of random input variables, the estimate value of random output, namely the expectation, variance and other statistical information is determined. As a comparison, the Monte Carlo simulation is utilized to settle the PHPF problem and the results in examined case are similar to ones obtained with scheme proposed in this paper.

Index Terms-- Probabilistic harmonic power flow, 2m+1 point estimate method, smart distribution network, distributed generators, Monte Carlo simulation

I. INTRODUCTION

The DGs access to smart distribution network increasingly by their special advantages. However, a large amount of harmonic inject to grid with DGs integration which may influence the power quality and even bring troubles to the power system operation [1]. Traditional harmonic power flow could calculate the harmonic current or voltage distribution of power grid. However, the stochastic fluctuations of harmonics generated by DGs also lead to the random variation of harmonics distribution [2]-[3]. As a result, evaluation of the Probabilistic Harmonics Power Flow (PHPF) is playing an important part in analysis the operation and planning and design DGs in power system.

At present, a lot of works have been done on probabilistic power flow while only few researchers concentrate on PHPF problem. There are three main methods of dealing with uncertain problem such as Monte Carlo Simulation method (MCSM) [4], analytical method and approximation method. The MCSM can easily simulate various uncertain problems in power system, but large amount of experiments are needed so that it is treaded as accuracy verification standard of other probabilistic methods [5]. Semi-invariant method (SIM) is usually chosen as representative of analytical method. It is high computational efficiency [6] using mathematical assumptions to linearization of research problems, but tedious mathematical derivation should be needed. Point estimate method (PEM) accredited as representative of approximate methods taking use of approximate formula for calculating the statistical properties and variables. The analytical expression between input random variables and output random variables is avoided. Specifically, 2m PEM was used to deal with probabilistic fundamental power flow calculation (PFPF) in [7]-[11] and probability of low frequency oscillation stability in [12]. However, 2m PEM ignores the higher order moments of the random variable so that the accuracy is poor. In [13], 2m+1 PEM was applied to solve the problem of PFPF and the simulation results show that 2m+1 PEM is more accurate than 2m PEM.

This paper describes a probabilistic approach to analyze the PHPF problem when DGs grid-tied. The presented scheme transferred an uncertain problem to a deterministic problem according to the 2m+1 PEM. Compared to MCSM, the proposed method guarantees substantial reductions of computational efforts and high accuracy in the output random variables characterization. Actual network is applied to verify the proposed algorithm with validity and high efficiency.

This paper is organized as follows. First, the conventional harmonic power flow is introduced. And then, the proposed method for PHPF is explained. Next, the detailed procedures are presented. Finally, the IEEE-14bus test system is applied to show efficiency of the proposed method.

II. HARMONIC POWER FLOW

Different from fundamental power flow based on power balance equation, the current balance equation is used to calculate harmonic power flow. The harmonic node admittance matrix $Y_h$ is usually adopted and express branch harmonic admittance values and the network connection. Harmonic network equation is represented as:

$$I_h = Y_h U_h$$  

(1)
Where \( I_h \) is the nodal current injection vector, \( U_h \) is the network admittance matrix at frequency \( h \) and \( \mathbf{U}_h \) the nodal voltage vector. Harmonic power flow is the basis of analyzing the PHPF. Considering the coupling relationship between fundamental power flow and harmonic power flow, the impact of the former to the latter is greater, so fundamental current and harmonic current of source are respectively:

\[
I_1 = g_1(U_1) \tag{2}
\]

\[
I_n = g_n(U_1, U_2, L), n = 2, 3, L \tag{3}
\]

Firstly, a fundamental power flow calculation should be conducted and get \( U_1 \) by Equation (2). Supposing harmonic voltage in Equation (3) is zero at all frequencies, the initial value of injected harmonic current can be obtained. Then nodal voltage could be calculated by Equation (1). Bringing the values to Equation (3), harmonic current correction value of harmonic source at every frequency could be obtained. If \( \Phi_1 \) is termed to be the 3\( n \) scheme, there are \( \Phi_2, \Phi_3, \Phi_4, \ldots \) scheme. Considering the coupling relationship between voltage vector. Harmonic power flow is the basis of analyzing network admittance matrix at frequency \( f \). Let \( 12(1, 2, \ldots, n) \) of each random variable are evaluated. According to the average order moments is utilized to evaluate probability density function \( f \). The 2M+1 Point Estimate Approach Theory

A. The 2M+1 Point Estimate Approach Theory

Point estimate is an effective method to solve the function \( Y = h(X) \) with n-dimensional random variable \( X \) and it is put forward by Hong [13] on the basis of Rosenblueth’s research [14]. The function \( Y = h(X) \) is Taylor expanded and the high order moments is utilized to evaluate probability density of \( Y \). Let \( X = (X_1, X_2, \ldots, X_n) \) and \( m \) estimate points \( x_{i,k} = (k = 1, 2, \ldots, m) \) of every random variable \( X_i \) are evaluated. According to the average \( \mu_i \) and variance \( \sigma_i \) of each random variable \( X_i \), estimate points is as follow:

\[
x_{i,k} = \mu_i + \xi_{i,k} \sigma_i \tag{4}
\]

Where \( \xi_{i,k} \) is coefficient of position. If the probability of \( X_i \) taking \( x_{i,k} = \omega_{i,k} \), there is

\[
\sum_{k=1}^{m} w_{i,k} = \frac{1}{n} \tag{5}
\]

\[
\sum_{j=1}^{n} \sum_{k=1}^{m} w_{i,k} = 1
\]

The central moment and standardized central moment are calculated as follow:

\[
M_{i,j} = \frac{(x_{i,1} - \mu_{i,k})^j + (x_{i,2} - \mu_{i,k})^j + \ldots + (x_{i,N} - \mu_{i,k})^j}{N} \tag{6}
\]

\[
\lambda_{i,j} = \frac{M_{i,j}}{(\sigma_i^j)}^j, j = 1, 2, 3, \ldots \tag{7}
\]

Where \( \lambda_{i,3} \) and \( \lambda_{i,4} \) is called skewness and kurtosis coefficient respectively. Using Taylor series, expanding the function \( Y = h(X) \) at mean value of \( X_i (i = 1, \ldots, n) \) and utilizing \( \lambda_{i,j} \) to estimate \( Y \) with \( m \) estimate points, there is

\[
\sum_{k=1}^{m} w_{i,k} (\xi_{i,k})^j = \lambda_{i,j}, i = 1, 2, \ldots, n, j = 1, 2, \ldots, 2m - 1 \tag{7}
\]

If \( m \) equal to three, namely, three estimation points could be adopted by each random variable \( X_i \). Assuming that one estimate point of the three is \( \mu_i \), make use of Equation (5) to calculate iteratively until it satisfies a given iteration convergence accuracy.

III. 2M+1 POINT ESTIMATE APPROACH FOR PROBABILISTIC HARMONIC POWER FLOW

A. The 2M+1 Point Estimate Approach Theory

Point estimate is an effective method to solve the function \( Y = h(X) \) with n-dimensional random variable \( X \) and it is put forward by Hong [13] on the basis of Rosenblueth’s research [14]. The function \( Y = h(X) \) is Taylor expanded and the high order moments is utilized to evaluate probability density of \( Y \). Let \( X = (X_1, X_2, \ldots, X_n) \) and \( m \) estimate points \( x_{i,k} (k = 1, 2, \ldots, m) \) of every random variable \( X_i (i = 1, 2, \ldots, n) \) are evaluated. According to the average \( \mu_i \) and variance \( \sigma_i \) of each random variable \( X_i \), estimate points is as follow:

\[
E(Y') = \sum_{i=1}^{n} \sum_{k=1}^{m} w_{i,k} \left[ h(\mu_1, \mu_2, \ldots, x_{i,k}, \ldots, \mu_n) \right]^l \tag{9}
\]

When \( l \) equals one, \( E(Y') \) is mean value of \( Y \) and When \( l \) equals two, standard deviation of \( Y \) is:

\[
\sigma_Y = \sqrt{E(Y^2) - E(Y)^2} \tag{10}
\]

Although 3n estimation points are constructed, there are \( n \) points corresponding to the same estimated point vector \( (\mu_1, \mu_2, \ldots, \mu_n) \) which led to calculation of \( Y \) once only. Therefore, the 3n scheme is transformed into the 2n+1 scheme.
B. The C type Gram-Charlier series for Probability Density Function

For probability density function (PDF) approximation problem in power system, Gram-Charlier series of A type is adopted because is simple and convenient. It is defined as follow [15]

$$f_X(x) = f_0(x) \left[1 + \sum_{i=1}^{\infty} \frac{\lambda_i H_i(x)}{i!}\right]$$  \hspace{1cm} (11)

Where $\lambda_i$ is series expansion coefficient, $H_i$ is the $i$th Hermite Orthogonal Polynomials and $f_0(x)$ is standard normal distribution PDF.

But only when the random variables of third-order and fourth-order central moment satisfy certain ranges, the type A Gram-Charlier series is feasible. Otherwise, probability distribution is valid and value of approximate PDF may be the negative, as a result, does not meet the basic axioms of probability. In view of the fact that the shortage of type A Gram-Charlier series is introduced to approximate PDF of random variables.

$$f_X(x) = \frac{\exp\left[\sum_{i=1}^{\infty} \frac{\gamma_i H_i(x)}{i!}\right]}{\int \exp\left[\sum_{i=1}^{\infty} \frac{\gamma_i H_i(x)}{i!}\right] dx}$$  \hspace{1cm} (12)

Where $\gamma_i$ is series expansion coefficient. It can be seen from this Formula (12) that exponential form is used to ensure the approximated value of PDF being greater than zero and the integration of PDF within integral domain identically equaling to one. Moreover, the advantages of type C series not only lies high accuracy of numerical results for normal distribution, but also satisfied distribution of deviating from the normal distribution.

C. The Harmonic Model of DGs

There are three ways of DGs accessing to grid, that is, synchronous generator, induction generator and power electronics inverter. Since inverters are adopted by most wind/photovoltaic and other renewable distributed power to get grid-tied, this article will consider harmonic deriving from power electronic inverters.

Analysis on the principle of three-phase current source inverter, the Fourier expansion of output line current is

$$i_l = \frac{2\sqrt{2}}{\pi} i_0 \left(\sin \alpha - \frac{1}{5}\sin 5 \alpha - \frac{1}{7}\sin 7 \alpha + \frac{1}{11}\sin 11 \alpha + ... + \frac{1}{h}\sin h \alpha...\right)$$  \hspace{1cm} (13)

Where the $i_0$ contains $6k \pm 1(k = 1, 2, 3, ...)$ = 5, 7, 11, ..., $h$. It is seen from Equation 13, harmonic current ratio of the $5^{th}, 7^{th}, 11^{th}, 13^{th}...h^{th}$ is 20%, 14.29%, 9.09%, 7.69%,...,$\frac{1}{h}$% respectively.

IV. PROCESS OF 2M+1 PEM FOR PHPF

The essence of calculating PHPF is to solve the power load equation containing random parameters. Among them, the input random variables are harmonic current injection and the output random variables include harmonic voltage of each node and branch harmonic current. To solve power flow equation, expectation, variance or probability distribution of input random variables should be made to obtain expectation, variance or PDF of output random variables.

To derive distribution ranges of harmonic voltage of each node, calculation method based on 2m+1 PEM is proposed in this paper and section below illustrates the steps.

1) Calculate the expectation and variance of each data sequence according to n sequences of harmonic current. Three estimate points and the corresponding probability values could be obtained by 2m+1 PEM.

2) Take one of the harmonic current amplitude as estimate value and the others as mean value $f(\mu_i, \mu_j, ...)$ .

3) Get harmonic voltage value in step 2 and do multiplication tables with probability values in step 1, add up and accumulate. That is

$$E(y) = \sum_{i} \sum_{j} p_{i,j} \left[f(\mu_i, \mu_j, ...)ight]$$

4) Squaring the harmonic voltage value in step 3 and multiplication tables with probability values in step 1, add up and accumulate. That is

$$E(y^2) = \sum_{i} \sum_{j} p_{i,j} \left[f(\mu_i, \mu_j, ...)^2\right].$$

5) Combine step 4 and step 5 and variance of branch harmonic current can be derived by Formula (10).

V. CASE STUDY

A. Example model

In order to consider PHPF of new energy integration to grid, the IEEE-14 example is adopted to verify the proposed method. The bus data and branch data based on fundamental power flow refer to reference [16], the generator output and load distribution based on harmonic power flow refer to reference [17]. Take wind power as the example of new energy integration, assuming that wind farm is connected to the system by bus No.4 while the bus NO.1 is the balance bus. The grid-connected DFIG is adopted by wind farm and power-generating capacity is 864 KW, nominal voltage is 690 V, the parameters of DFIG refer to [18].

The random model of harmonic current injected by accessing of new energy is as discussion in section 3. Assuming that each order of harmonic current is independent and amplitude of harmonic current satisfies normal distribution while the phase of harmonic current is ignored temporarily. Actually random amplitude components of harmonic current usually fluctuate up and down around a
fixed value. The $2m+1$ PEM and MCSM (1000 sampling) are adopted to calculate PHPF.

This paper set the definite value of harmonic current as the amplitude of harmonic current component derived from calculation of harmonic power flow. That is, the expectation of the harmonic current component is equal to the amplitude of harmonic current component, and the variance equals to 10% of expectation. The value of random harmonic current injection is in Table I.

| TABLE I. RANDOM INPUT VARIABLES MODEL OF HARMONIC SOURCE |
|---|---|---|---|---|
| Ih% | 5th | 7th | 11th | 13th | 15th |
| Mean | 0.1394 | 0.0997 | 0.0638 | 0.0538 | 0.0407 |
| Variance | 0.00122 | 0.00064 | 0.000243 | 0.00166 | 0.00106 |

B. Result analysis

The $2m+1$ PEM for PHPF in this paper is used to calculate harmonic voltage stochastic behavior of a single node and the entire network. And the PDF of harmonic current is compared with the MCSM. Result is shown in Figure 1:

![Figure 1. PDF distribution curves of Node 1 harmonic voltage](image)

The dotted line and solid line showing in Figure 1 express PDF distribution curves of Node 1 by using $2m+1$ PEM and MCM respectively. AS seen in Figure 1, the dotted line and solid line are basically identical so that the proposed method is feasible.

1) From the entire network

a) The closer electrical distance between non-harmonic source node and the distributed power harmonic source is, the greater the CP95% of voltage amplitude of each harmonic component will be, Conversely, the smaller.

CP95% is 95% of cumulative probability of harmonic voltage amplitude corresponding to the value of the random variable. The CP95% of distribution of harmonic voltage amplitude is shown in Figure 2 (only take the 13th harmonic voltage ratio for example).

![Figure 2. CP95% distribution of the 13th harmonic voltage ratio](image)

As illustrated in Figure 2, from the nodes NO.3 and NO.5 whose electrical distance is relatively close to the node NO.4 which is connected to wind farm, CP95% of harmonic voltage amplitude increase gradually. Meanwhile, the CP95% value of harmonic source is greater obviously to those non-harmonic source, namely, the nearer the harmonic source in electric distance, the greater the possibility of harmonic voltage amplitude upper limit.

b) When electrical distance between non-harmonic source node and the distributed power harmonic source is closer, the CP95% of total harmonic distortion (THD) becomes higher. Conversely, the smaller. Such as node No.6, 8 and 14.

![Figure 3. CP95% of total harmonic distortion for each node](image)

It is shown in Figure 3, CP95% of THD of the node NO.4 connected with wind farm exceeds the national standard limit 5% while which of the other nodes is less than 5%. In addition, the THD of each node 9, 10, 11, 12, 13, 14 is less than 4%. However, THD value of node NO.3 and 5 is still high. As a result, the random fluctuations of harmonic sources may make important influence on THD of each node in this paper. Although the calculated results show that CP95% of THD can satisfy the requirement of power quality (except harmonic injection point), but considering the actual runtime, the injected harmonic caused by distributed generation is different from simulation computation. Study on harmonic randomness which is caused by new energy accessing procedure to grid is of great significance.

2) From the single node
Taking node NO.1 for example and the PDF curves of harmonic voltage THD for each order are shown in Figure 4.

Figure 4. PDF and CDF curves of THD on Node 1

Cumulative distribution curve of harmonic voltage THD on Node 1 equals to 95%, the corresponding value of THD is 0.0473, which means the possibility of harmonic voltage THD range equaling to 95%. That is, probability that harmonic voltage distortion rate fluctuates upper limit to 4.73% is 95%.

VI. CONCLUSION

A probabilistic method based on 2m+1PEM for PHPF has been proposed to evaluate the distribution of harmonic when DGs accessing to grid. This method takes into account the uncertainty including amplitude of harmonic current injection source. The proposed scheme avoids establishing analytic form relationship between input random variables and output random variables. Type C Gram-Charlier series is introduced to approximate the PDF of harmonic nodal voltage. The proposed scheme is compared with the Monte Carlo to verify the dependence between input nodal powers. The results, in the examined cases, are close to the ones obtained with a Monte Carlo procedure. The CP95% of harmonic voltage is obtained by IEEE 14 buses simulation which has statistical significance.

ACKNOWLEDGMENT

The authors gratefully acknowledge the contributions of T. Edison, G. Westinghouse, N. Tesla, A. Volta and A. Ampere to the electric power industry. The authors gratefully thank the contributions and supports from National Natural Science Foundation committee of China.

REFERENCES